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Higher-Order Loss Processes and the Loss Penalty of Multimode Operation

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Pulse spreading caused by the different group velocities of the guided modes of a multimode waveguide can be reduced by providing intentional coupling between the modes. Coupling among the guided modes inevitably leads to radiation losses. This loss penalty is discussed for two types of loss processes. We consider that the highest-order mode loses power by second-order coupling to the continuous spectrum of radiation modes. We also consider a loss process that is caused by nonresonant coupling of guided modes to lossy neighboring modes. Both loss processes can cause a substantial loss penalty. However, the loss penalty can always be reduced by limiting the intentional coupling to fewer of the guided modes, allowing the highest-order modes to die out. The discussion is based on a slab waveguide model.

I. INTRODUCTION

Higher-order loss processes have been discussed in a previous paper.[1] The idea of loss processes of different orders is based on perturbation theory. Two modes of a dielectric waveguide are coupled if their propagation constants obey the relation[1]

$$|\beta_\nu - \beta_\mu| = m\phi. \tag{1}$$

ϕ is the mechanical frequency of the Fourier spectrum of the coupling function; m is a positive integer that specifies the order of the coupling process. If $m = 1$, mode ν is coupled to mode μ by a first-order process, $m = 2$ indicates a second-order process, etc. For small values of a/λ_o, (a is the Fourier amplitude that belongs to the mechanical frequency ϕ; λ_o is the free space wavelength of the light in the waveguide) the coupling strength is proportional to $(a/\lambda_o)^m$ so that the coupling decreases with increasing order of the coupling process. If mode ν represents a guided mode, mode μ may either be a guided or a radiation mode. In the latter case, mode ν loses power by radiation. An explanation of the coupling process in terms of diffraction gratings is given in Ref. 1.

There is a different loss process that cannot be understood in terms of higher-order grating lobes. Consider two guided modes. Mode 2 is inherently lossless, while mode 1 suffers high loss. If we couple these two modes by means of a first-order process, a large amount of loss will be transferred from the lossy mode to the hitherto lossless neighbor. However, even if we couple these two modes by means of a sinusoidal coupling function the mechanical frequency of which does not satisfy (1) for any integer m, some loss will be imparted from the lossy mode to the inherently lossless mode. If both modes were lossless, no significant amount of power would be interchanged among them if (1) is not satisfied for $m = 1$ (or any other integer). We call such a coupling process "nonresonant coupling." The small amount of power that flows momentarily from mode 1 to mode 2 is returned in the next instant because the phase relationship required for continuous power flow from one mode to the other does not exist. However, if mode 1 is lossy, mode 2 transfers a small amount of power to mode 1 (even via the nonresonant coupling process) that can not be returned since some of the power is already dissipated in the lossy mode 1. This nonresonant coupling process has the effect of imparting some of the high loss of one mode to a neighboring mode. The attenuation coefficient that results is derived in the Appendix.

In this paper, we calculate the loss penalty that stems from intentional mode coupling in a multimode slab waveguide caused by these higher-order processes. Reference 2 presents the theory of pulse propagation in multimode waveguides in the presence of first-order coupling between the guided modes. The purpose of the coupling is to reduce pulse distortion.[3] It was pointed out that it is possible to couple all the guided modes by a first-order process without causing first-order radiation losses.[2] This possibility arises from the fact that the modes of a slab waveguide are arranged in β space, such that the spacing between neighboring modes increases with increasing mode number. Because of the coupling law (1) with $m = 1$, it is possible to couple all the guided modes, except mode N, by providing a spectrum of mechanical frequencies that has a proper Fourier component for coupling at least the nearest neighbors of all the modes. However, mode N is not coupled to mode $N - 1$ if the Fourier spectrum has an abrupt cutoff so that no mechanical frequency exists that satisfies the relation

$$\beta_{N-1} - \beta_N = \phi. \tag{2}$$

Residual losses result from the fact that it is unrealistic to assume a

Fourier spectrum with an abrupt cutoff. Residual coupling between mode $N - 1$ and mode N is then possible via the tail of the Fourier spectrum. Mode N is necessarily coupled by first-order processes to the continuous spectrum of radiation modes so that mode $N - 1$, being coupled to mode N, suffers loss which causes power loss to the entire ensemble of coupled guided modes. We must now consider the effect of higher-order processes. Even with a Fourier spectrum with perfectly abrupt cutoff, mode $N - 1$ is coupled to the spectrum of radiation modes by second- and higher-order processes. Assuming small amplitudes for the Fourier coefficients, we neglect processes of third- and higher-order and discuss radiation losses caused by the second-order process. We shall see that substantial losses can result even via the second-order loss mechanism. However, luckily, we can readjust the intentional coupling between the guided modes to prevent first-order coupling not only to mode N but also to mode $N - 1$. The distance (in β space) between mode $N - 2$ and the continuum of radiation modes is then greater than 2ϕ so that the second-order loss process is no longer possible. The uncoupled modes (uncoupled from the remaining guided modes) lose power by being coupled to the radiation field. Mode N loses power very rapidly because it is coupled by means of a first-order process. Mode $N - 1$ loses power by means of a second-order process. If the loss caused by coupling of the guided modes to mode $N - 1$ was bothersome, its loss is certainly sufficient to prevent pulse distortion by power flowing along in this mode. It is thus clear that radiation losses can be reduced by limiting the intentional coupling to the lower-order guided modes leaving a few of the higher-order modes to die out because of their high radiation losses.

In a similar manner, nonresonant coupling between the lossy mode N (coupled by a first-order process to radiation modes) and the neighboring guided modes $N - 1$, $N - 2$, etc., influences the loss behavior of the intentionally coupled guided modes. The loss penalty caused by this nonresonant coupling mechanism is considered separately from the higher-order loss process mentioned earlier in order to assess the separate influence of each mechanism. Again, it is advantageous to uncouple some of the higher-order modes from the lower-order guided modes since the nonresonant coupling process decreases in strength with increasing distance (in β space) of the guided modes from the lossy mode N.

II. SUMMARY OF COUPLED POWER THEORY

The coupled power theory presented in Ref. 2 was based on the stochastic partial differential equation for the average power of the modes

$$\frac{\partial P_\nu}{\partial z} + \frac{1}{v_\nu}\frac{\partial P_\nu}{\partial t} = -\alpha_\nu P_\nu + \sum_{\mu=1}^{N} h_{\nu\mu}(P_\mu - P_\nu). \tag{3}$$

The power loss coefficient α_ν , for mode ν with group velocity v_ν , incorporates heat losses as well as radiation losses. However, heat losses will be ignored in our present discussion. For slab waveguides with random core-cladding interface perturbations, the coupling coefficient assumes the form

$$h_{\nu\mu} = \frac{n_1^2 \mathrm{k}^2 \sin^2 \theta_\nu \sin^2 \theta_\mu}{2d^2\left(1 + \frac{1}{\gamma_\nu d}\right)\left(1 + \frac{1}{\gamma_\mu d}\right) \cos \theta_\nu \cos \theta_\mu} F(\beta_\nu - \beta_\mu). \tag{4}$$

The mode angle θ_ν is defined in terms of the refractive index n_1 of the core, the free space propagation constant k and the propagation constant β_ν of the νth mode.

$$\cos \theta_\nu = \frac{\beta_\nu}{n_1 \mathrm{k}}. \tag{5}$$

The parameter γ_ν appearing in (4) is (n_2 = cladding index)

$$\gamma_\nu = (\beta_\nu^2 - n_2^2 k^2)^{1/2} \tag{6}$$

and d is the slab half width. The function $F(\phi)$ is the ensemble average of the square of the Fourier transform of the core-cladding interface function. It will be referred to as the "power spectrum." For the purpose of this paper, we assume that $F(\phi)$ is constant from zero to the cutoff value ϕ_c of ϕ. For $\phi > \phi_c$ we assume that $F(\phi) = 0$.

For sufficiently large values of z, we obtain the following approximate solution of (3)[2]

$$P_\nu(z, t) = \frac{2\tau}{\Delta t}\mathrm{k}_1 B_{\nu o}^{(1)} e^{-\alpha_o^{(1)} z} \exp\left[-\left(\frac{t - z/v}{\Delta t/2}\right)^2\right], \tag{7}$$

with the full width of the Gaussian pulse given by

$$\Delta t = 2(\tau^2 + 4\alpha_2^{(1)} z)^{1/2}. \tag{8}$$

The input pulse is determined by its half width τ and amplitude G_ν

$$P_\nu(0, t) = G_\nu \exp\left(-\frac{t^2}{\tau^2}\right). \tag{9}$$

The coefficient k_1 is given by

$$\mathrm{k}_1 = \sum_{\nu=1}^{N} G_\nu B_{\nu o}^{(1)}. \tag{10}$$

$B_{\nu o}^{(1)}$ and $\alpha_o^{(1)}$ are defined as the first eigenvector and eigenvalue of an eigenvalue problem the details of which can be found in Ref. 2. The parameter $\alpha_2^{(1)}$ appearing in (8) is the second-order perturbation of the eigenvalue:

$$\alpha_2^{(1)} = \sum_{j=2}^{N} \frac{\left\{\sum_{\nu=1}^{N}\left(\frac{1}{v_\nu} - \frac{1}{v}\right)B_{\nu o}^{(1)}B_{\nu o}^{(j)}\right\}^2}{\alpha_o^{(j)} - \alpha_o^{(1)}}. \tag{11}$$

The superscript j identifies $B_{\nu o}^{(j)}$ and $\alpha_o^{(j)}$ as the jth eigenvector and eigenvalue of the eigenvalue problem; v is the average group velocity. It is convenient to use the parameter ($\tau \to 0$ is assumed)

$$R = \frac{\Delta t}{\Delta T} = \frac{4\sqrt{\alpha_2^{(1)}}}{\left(\frac{1}{v_N} - \frac{1}{v_1}\right)\sqrt{L}} \tag{12}$$

that was introduced in Ref. 2 as a measure of the improvement of the pulse distortion of coupled modes compared to the uncoupled case for a guide of length L. ΔT is the length in time covered by the signal arriving in the many uncoupled modes traveling with different group velocities. It is desirable to make R as small as possible by means of coupling between the guided modes.

Finally, we quote the formulas for the power loss coefficients. From Ref. 1 we obtain for the second-order loss attributable to the second-order grating lobe

$$\alpha = \frac{a^4\kappa^2(n_1^2 - n_2^2)\mathrm{k}^2}{32\beta_g(\sigma_{-2} + \rho_{-2})d}\left[4\sigma_{-1}^2 + \sigma_{-2}\rho_{-2} + (\gamma - 2\gamma_{-1})^2\right]. \tag{13}$$

In addition to the parameters already defined earlier, we have

$$\beta_g = \text{propagation constant of the guided mode,}$$

$$f(z) = a \sin \phi z, \text{ core-cladding interface distortion,} \tag{14}$$

$$d = \text{slab half width,}$$

$$\kappa = (n_1^2\mathrm{k}^2 - \beta_g^2)^{\frac{1}{2}}, \tag{15a}$$

$$\sigma_{-1} = [n_1^2\mathrm{k}^2 - (\beta_g - \phi)^2]^{\frac{1}{2}}, \tag{15b}$$

$$\sigma_{-2} = [n_1^2\mathrm{k}^2 - (\beta_g - 2\phi)^2]^{\frac{1}{2}}, \tag{15c}$$

$$\gamma_{-1} = [(\beta_g - \phi)^2 - n_2^2\mathrm{k}^2]^{\frac{1}{2}}, \tag{15d}$$

$$\rho_{-2} = [n_2^2\mathrm{k}^2 - (\beta_g - 2\phi)^2]^{\frac{1}{2}}, \tag{15e}$$

$$\gamma = [\beta_g^2 - n_2^2\mathrm{k}^2]^{\frac{1}{2}}. \tag{15f}$$

The mode power loss coefficient α_s for nonresonant coupling is derived in the Appendix.

$$\alpha_s = \frac{1}{2} \sum_{\substack{\mu=1 \\ \mu \neq s}}^{N} \frac{| K_{\mu s} |^2 a^2}{(\alpha_\mu/2)^2 + (\beta_\mu - \beta_s - \phi)^2} \alpha_\mu . \tag{16}$$

The power loss coefficient of the μth mode is α_μ, $| K_{\mu\nu} |^2$ is the factor of $F(\phi)$ in (4) and a is defined in (17).

III. POWER SPECTRUM OF A SINE WAVE WITH RANDOM PHASE

The second-order radiation loss formula and the loss formula for nonresonant coupling to lossy modes were derived for sinusoidally deformed core-cladding interfaces. We expect that these results remain valid, at least approximately, even if the sinusoidal interface variation has a random phase. This random phase assumption is important to ensure the validity of certain averaging procedures that were involved in deriving equation (13). A purely sinusoidal interface variation with constant phase is not likely to occur in practice. For this reason, we derive the relation between the amplitude a of the sinusoidal interface variation, and the power spectrum $F(\phi)$ of the Fourier spectrum of the sinusoidal process with random phase. We are using the idea of a sinusoidal core-cladding interface distortion and the concept of a flat power spectrum of this function with a definite cutoff "frequency" as though they were compatible with each other. It appears possible that a suitable probability distribution for the random phase of the sinusoidal process could be found that would approximate the desired flat power spectrum. However, we make no effort to investigate the compatibility of these ideas, and use them simultaneously in order to gain an order of magnitude estimate of the loss penalty from higher-order loss process that results from intentional, ideal coupling between the guided modes.

In order to establish the desired relation between the amplitude a of the sinusoidal function $f(z)$, describing the core-cladding interface irregularity and the power spectrum of this function, we introduce

$$f(z) = a \sin [\phi z + \psi(z)], \tag{17}$$

with the mechanical frequency ϕ, amplitude a, and random phase $\psi(z)$. The power spectrum is related to $\langle f^2 \rangle$ by the following equation

$$\langle a^2 \sin^2 (\phi z + \psi) \rangle = \frac{1}{\pi} \int_0^\infty F(\phi') \, d\phi' . \tag{18}$$

The symbol $\langle\ \rangle$ indicates an ensemble average. We assume (without justification) that the power spectrum has the shape

$$F(\phi') = \begin{cases} \hat{F} = \text{const} & \text{for} \quad 0 \leqq \phi' \leqq \phi_c \\ 0 & \text{for} \quad \phi' > \phi_c \, . \end{cases} \tag{19}$$

Since the ensemble average of the square of the sine function is 1/2, we obtain from (18) and (19)

$$\hat{F} = \frac{\pi a^2}{2\phi_c}. \tag{20}$$

The assumption of the flat power spectrum with cutoff, (19), ensures that to first order of perturbation theory no power loss occurs provided that ϕ_c is chosen such that

$$|\beta_{n-1} - \beta_n| > \phi_c \, . \tag{21}$$

All modes with mode number $\nu < n$ are coupled to each other, while no first-order coupling to the guided modes $N \geqq \nu \geqq n$ and to the radiation modes is possible. The residual losses that still exist are thus caused by the higher-order processes that are the object of our study.

We are using $\phi = \phi_c$ in the eqs. (15) through (16).

IV. DISCUSSION OF THE EFFECT OF SECOND-ORDER LOSS

We are now ready to calculate the loss penalty that has to be paid for coupling the guided modes with a coupling function the power spectrum of which is given by (19). We assume that the loss mechanism is second-order coupling of mode N (the highest-order guided mode) to the continuous spectrum of radiation modes. We are ignoring the fact that the highest-order mode is usually coupled to the radiation modes by means of a first-order process. However, our assumption is not unrealistic since we can regard mode N as the last of the guided modes that is still coupled to all the other modes, but which is not the mode nearest to the continuous spectrum of radiation modes. If, for example, mode N is to be taken as being the next to last guided mode, it need not be coupled to the last mode by a first-order process, but can itself be coupled to the continuous spectrum of radiation modes by means of second-order coupling. We are thus using the loss coefficient of equation (13) for α_N appearing in (3) while setting $\alpha_\nu = 0$ for $\nu \neq N$.

Figure 1 shows the loss penalty for the 3, 5, 10 and 20 mode case. Since our model is a slab waveguide, the guided modes are the TE modes of a slab. Both core-cladding interfaces of the slab are considered

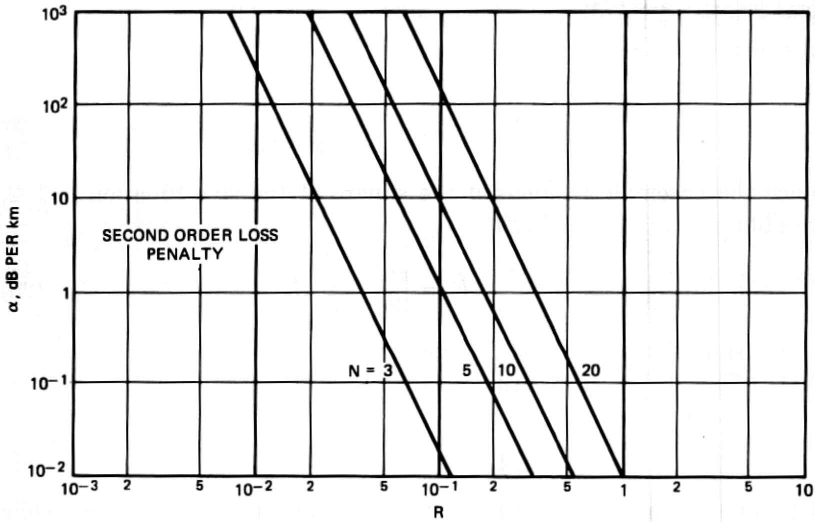

Fig. 1—Loss penalty caused by second-order radiation losses of mode N. α is the power loss coefficient. R is the improvement factor (ratio of pulse width of coupled modes to pulse width of uncoupled modes). N is the number of modes.

to be distorted with the power spectrum of the distortion function given by (19) and (20). We assume in our model that the index ratio of core-to-cladding index is $n_1/n_2 = 1.01$ with $n_1 = 1.5$. The values of kd are

$$\text{k}d = 16.5 \quad \text{for 3 modes}$$

$$\text{k}d = 35 \quad \text{for 5 modes}$$

$$\text{k}d = 70 \quad \text{for 10 modes}$$

$$\text{k}d = 145 \quad \text{for 20 modes.}$$

The loss is the steady-state loss per kilometer. We thus assume implicitly that the steady-state distribution is reached, and that the loss is the decrease in power of the steady-state power distribution. (See Ref. 4 for an explanation.) The steady-state loss is plotted as a function of the improvement factor R defined by (12). $R = 0.1$, for example, means that the width of the pulse carried by the coupled guided modes is ten times narrower than it would be in the absence of coupling. The loss penalty increases rapidly with the number of modes. If the third mode of a total of three modes is coupled by the second-order process to the radiation field, an improvement by ten, $R = 0.1$, causes very little radiation loss. However, we see from Fig. 1 that the loss penalty for the 20-mode case is already more than 100 dB/km. This shows that even the losses caused by second-order coupling of the highest-order mode to the radiation field can cause intolerably high losses if the coupling between the guided modes is strong enough for R to reach $R = 0.1$.

However, to keep the proper perspective, it is important to note that the loss caused by this second-order mechanism would be reduced to zero simply by uncoupling the highest-order mode, and restricting the coupling between the guided modes [by reducing the width of the spectral distribution (19)] to mode 1 through $N - 1$. There are still other losses to contend with. One of these mechanisms will be discussed in the next section. However, second-order losses can be rendered harmless by this device.

It is of interest to know how large an amplitude of the sinusoidal core-cladding interface distortion with random phase is required to cause a given improvement factor R. This question is answered by Fig. 2. The curves of this figure extend below the value $R = 1$, since values of $R > 1$ are of no interest. They are also limited to values of $\mathrm{k}a < 1$, because for larger values of $\mathrm{k}a$ our perturbation theory becomes meaningless. The figure shows clearly that an improvement factor of $R = 0.1$ can only be reached for fairly large values of $\mathrm{k}a$. In the 20-mode case, we find $\mathrm{k}a = 1$ for $R = 0.1$ so that we are approaching the limit of applicability of the second-order perturbation theory used to derive the coupling coefficient (4) and the loss coefficient (13).

V. DISCUSSION OF THE EFFECT OF NONRESONANT COUPLING

We are now considering the nonresonant loss mechanism that led to eq. (16). We are using this equation in the following way. We assume

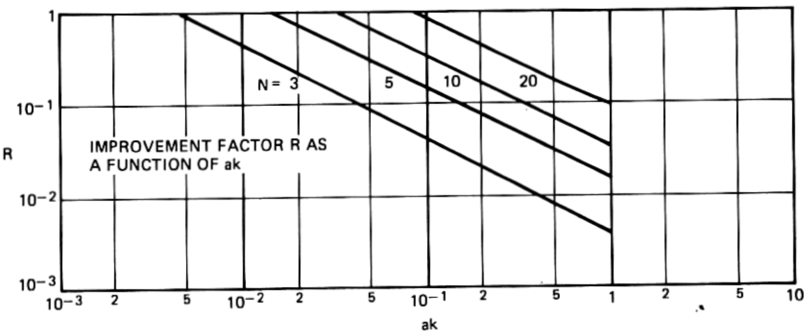

Fig. 2—Improvement factor R as a function of ak. (a = amplitude of sinusoidal core-cladding interface distortion, $\mathrm{k} = 2\pi/\lambda_o$).

that the highest-order mode, mode N, is coupled strongly by means of a first-order process to the radiation field. The loss coefficient for this case can be found in equation (63) of Ref. 1. Next, we consider the loss that is transmitted from this high-loss mode to its neighbors. We use eq. (16) to compute the losses of mode $N - 1$, $N - 2$, etc., successively substituting the loss value of each successive iteration to obtain the loss of the next lower mode. We stop at the last mode that is already coupled by a first-order process to the remaining guided modes. The loss penalty that results from coupling this mode to all the other guided modes is being considered here.

Figure 3 shows the loss coefficients for the 10-mode case. The curve on the extreme left is the loss coefficient of mode 10 that loses power via the first-order process to the radiation modes. The modes labeled $s = 9, 8, 7$, etc., suffer loss because of nonresonant coupling to mode 10. The different slopes of these two sets of curves is caused by the fact that the first-order loss process is proportional to $(ak)^2$ while the nonresonant losses are proportional to $(ak)^4$. For a given value of ak, the losses decrease rapidly with decreasing value of s. However, it is surprising how high the losses caused by nonresonant coupling are if $ak = 1$. Figure 4 shows the same data for the 20-mode case.

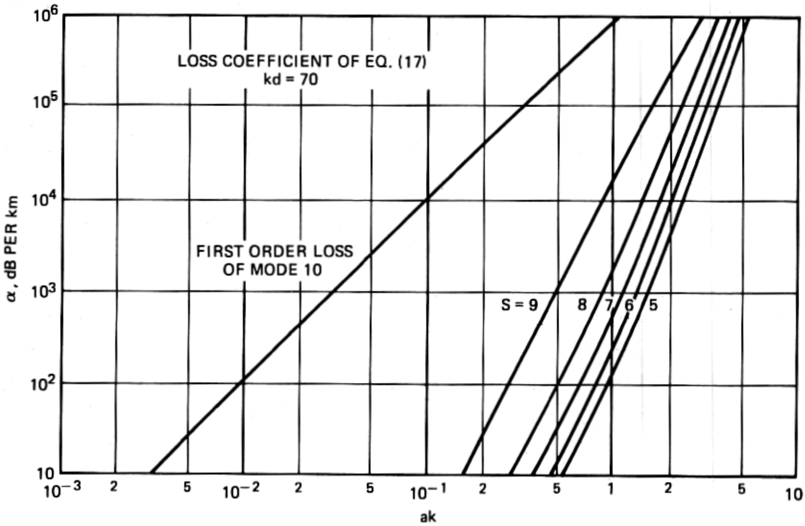

Fig. 3—Mode loss as a function of ak. s is the mode number. The loss is caused by nonresonant coupling of the modes $N - 1$, through mode s to the lossy mode N. The first-order loss of mode $N (N = 10)$ is the curve on the left of the figure.

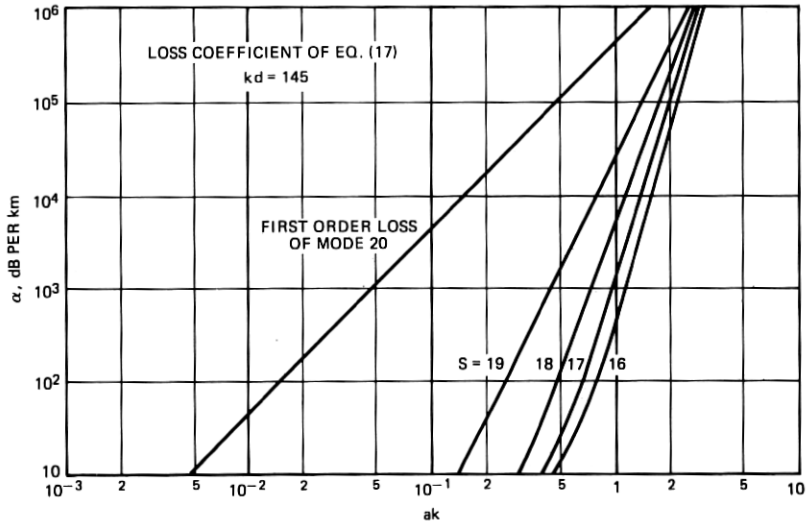

Fig. 4—Same as Fig. 3, with $N = 20$.

Figure 5 shows the loss penalty that results from nonresonant coupling of mode $N - 1$ to the lossy mode N, while modes 1 through $N - 1$ are coupled to each other by the resonant first-order process. The loss penalty is again plotted as a function of the improvement factor R. Figure 5 shows an interesting phenomenon. Whereas the curves for $N = 3$ and $N = 5$ are straight, the curves for $N = 10$ and $N = 20$ are bent. The reason for this difference in behavior can be explained if we consider the shape of the steady-state distribution of mode power P_ν versus mode number ν. All along the curves for $N = 3$ and $N = 5$, the steady-state power distribution is flat; that means we have equal power in all the modes. On that portion of the curve labeled $N = 10$ that is parallel to the curves with $N = 3$ and 5, we find also that equal power is carried by all the modes in the steady state. However, when the curve begins to bend over, we enter a region where the steady-state power distribution begins to change, favoring the lower-order modes. The loss penalty is correspondingly far less in that region than it would be if the original slope of the curve had been maintained. This is not surprising if we consider that only very little power remains, even in the steady state, in the higher-order modes that couple strongly to the lossy mode $N - 1$. By redistributing the steady-state distribution, the multimode waveguide manages to operate with lower losses. We thus

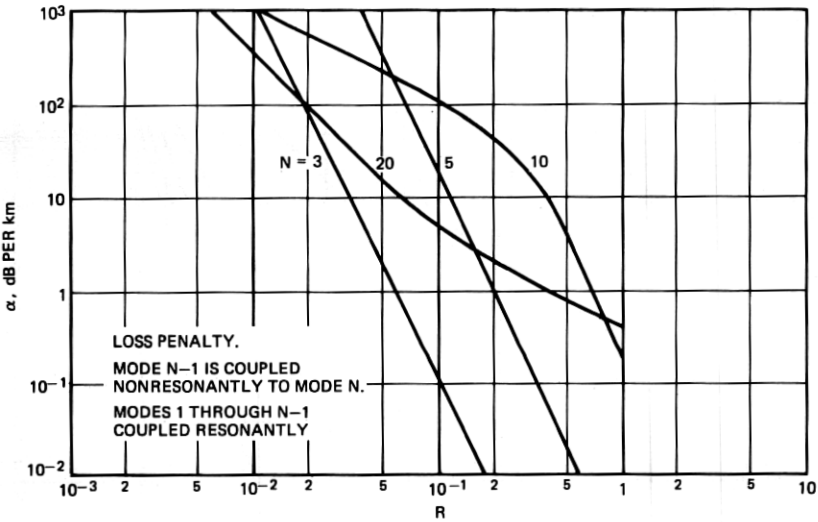

Fig. 5—Loss penalty caused by mode coupling among all $N - 1$ modes and nonresonant coupling of mode $N - 1$ to mode N. The independent variable is the improvement factor R.

find the paradox that, for equal values of R, the 5-mode guide can be lossier than the 10- and 20-mode guide. However, the improvement in the value of R is obtained not by stronger coupling of all the guided modes, but primarily by a reduction in the number of modes that still carry power.

The remaining figures show what happens if we couple fewer guided modes to each other allowing the higher-order modes to die out due to radiation losses. Figure 6 shows the 5-mode case with 4 and 3 guided modes coupled to each other. The improvement in the loss penalty that results from dropping mode 4 from the set of coupled guided modes is substantial.

The same behavior is shown for the 10-mode case in Fig. 7. Again it is apparent how much improvement in the loss penalty can be gained by dropping successively the higher-order modes from the set of coupled guided modes. Only the curve with $n = 9$ behaves anomalously. The change in slope can again be explained by the change in the steady-state distribution. The region with gentler slope corresponds to a steady-state distribution that no longer carries equal power in all the modes but favors the lower-order modes.

This tendency to flip from a steady-state distribution with equal

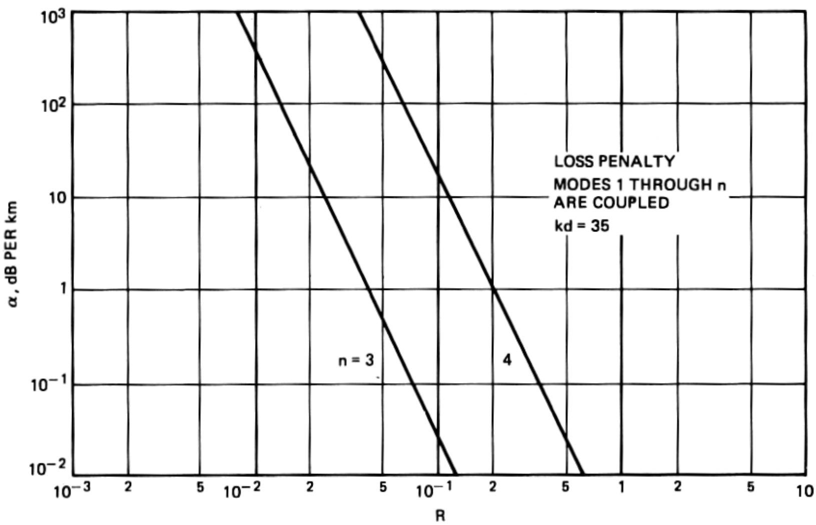

Fig. 6—Loss penalty caused by mode coupling of modes 1 through n and non-resonant coupling of modes n, $n + 1$, etc. to mode $N.N = 5$.

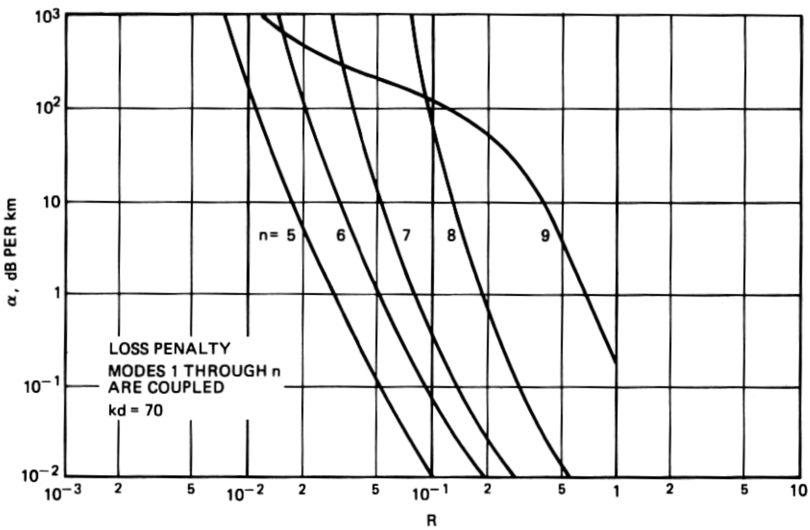

Fig. 7—Same as Fig. 6 for $N = 10$. The curve labeled $n = 9$ departs from the other curves because of a change in the steady-state power distribution.

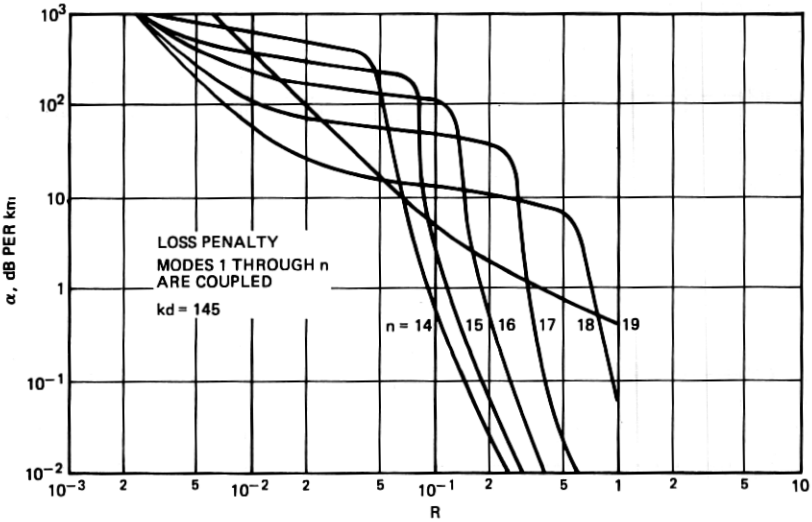

Fig. 8—Same as Figs. 6 and 7 with $N = 20$.

power in all the modes to one that favors lower-order modes is even more apparent in the 20-mode case shown in Figure 8. We also see in this figure that if we want to operate with an improvement factor of $R = 0.1$, and tolerate a loss of 1 dB/km we must uncouple 6 modes from the total of 20 modes allowing only the lowest 14 modes to couple among each other.

VI. CONCLUSIONS

We have studied the loss penalty that results from higher-order loss processes. We have considered two different cases. In both cases, we let most of the guided modes be coupled by a first-order resonant process. In the first case, we assumed that the highest-order mode is coupled to the radiation modes only by means of a second-order process. High losses can still result if we want to achieve a good pulse spreading reduction by means of strong coupling of the guided modes. The loss penalty increases very rapidly with increasing mode number for a fixed value of the improvement factor R. However, by limiting the coupling to one less guided mode, allowing the highest-order guided mode (or more accurately the two highest-order guided modes) to die out, the loss penalty from this second-order process disappears.

There are other processes that still cause a loss penalty even if we drop the two highest-order modes. The lossy modes impart some of their loss to their neighbors via a nonresonant coupling process. The loss penalty from this mechanism can still be high. Again it helps to limit the coupling to fewer of the lower-order modes by reducing the width of the power spectrum of the coupling function. By proper design of the coupling process, the loss penalty for a given improvement factor can be kept in tolerable limits. By uncoupling some of the higher-order modes, we pay an additional loss penalty in the transient before the steady-state distribution has established itself, provided that all modes are excited equally at the beginning of the waveguide.

Our results were obtained by using the model of the slab waveguide. However, they allow an estimate of the performance of the round optical fiber if we keep in mind that the total number of modes of the round fiber is the square of the mode number of the slab waveguide. The 10-mode case of the slab waveguide thus corresponds to a 100-mode round optical fiber. The members within each family of modes with equal circumferential field distribution (the same value of ν in $\cos \nu\phi$) are coupled among each other by diameter changes of the fiber core. Each family of this kind behaves similarly to the modes of the slab waveguide studied here. We are thus able to use the results of the slab waveguide to draw conclusions about the expected behavior of round optical fibers.

APPENDIX

Derivation of Equations (16)

As a starting point we use the coupled wave equations.[5]

$$\frac{da_\nu}{dz} = -i\bar{\beta}_\nu a_\nu + \sum_{\substack{\mu=1 \\ \mu\neq\nu}}^{N} c_{\nu\mu}(z)a_\mu. \tag{22}$$

The propagation constants $\bar{\beta}_\nu$ are assumed to be complex quantities, $c_{\nu\mu}$ are the coupling coefficients and a_ν are the mode amplitudes. With the slowly varying mode amplitudes A_ν defined by

$$a_\nu = A_\nu e^{-i\bar{\beta}_\nu z} \tag{23}$$

the system of coupled wave equations is transformed into the following form

$$\frac{dA_\nu}{dz} = \sum_{\substack{\mu=1 \\ \mu\neq\nu}}^{N} c_{\nu\mu}(z)A_\mu e^{i(\bar{\beta}_\nu - \bar{\beta}_\mu)z}. \tag{24}$$

We now assume that only one of the modes, mode s, is strongly excited at $z = 0$, while all the other mode amplitudes vanish initially

$$A_\nu(0) = 0 \qquad \nu \neq s. \tag{25}$$

In the vicinity of $z = 0$, we thus obtain approximately

$$\frac{dA_\nu}{dz} = c_{\nu s}(z) A_s e^{i(\bar{\beta}_\nu - \bar{\beta}_s)z} \quad \text{for} \quad \nu \neq s \tag{26}$$

and for $\nu = s$ (since $\bar{\beta}_s$ is assumed to be real we write $\bar{\beta}_s = \beta_s$)

$$\frac{dA_s}{ds} = \sum_{\substack{\mu=1 \\ \mu \neq s}}^{N} c_{s\mu}(z) A_\mu e^{i(\beta_s - \bar{\beta}_\mu)z}. \tag{27}$$

In analogy with the Wigner-Weisskopf method,[6] we next assume that the z dependence of the mode amplitude A_s is given by

$$A_s(z) = A_s(0) e^{-\frac{1}{2}\alpha_s z}. \tag{28}$$

The determination of the unknown constant α_s is the objective of the following calculation. Substitution of (28) into (26) and subsequent integration results in

$$A_\mu(z) = A_s(0) \int_0^z c_{\mu s}(x) e^{[i(\bar{\beta}_\mu - \beta_s) - (\alpha_s/2)]x} \, dx. \tag{29}$$

At this point it becomes necessary to specify the z dependence of the coupling function $c_{\nu\mu}(z)$. We want to determine the effect of nonresonant coupling but are, nevertheless, interested in the influence of a periodic coupling function. For simplicity it is convenient to assume that the coupling coefficients are of the following form

$$c_{\mu s}(z) = \frac{a}{\sqrt{2}} K_{\mu s} e^{-i\phi z}. \tag{30}$$

It is a well-established fact that the coupling coefficient can be decomposed into a constant part $K_{\nu\mu}$ times a function of z. If mode coupling is caused by core-cladding interface irregularities of dielectric waveguides, the z-dependent function describes directly the shape of the core-cladding interface deformation.[4] Ordinarily, we would expect to see a sine or a cosine function instead of the exponential function that appears in (30) provided that the core-cladding interface distortion is purely sinusoidal. But a sinusoidal function can always be decomposed into two exponential functions. We keep only one of these two terms. This approximation is justified if we consider near-resonant coupling. Only terms with small values of $\bar{\beta}_\nu - \bar{\beta}_\mu - \phi$ will be seen to give a substantial constribution. The term $\bar{\beta}_\nu - \bar{\beta}_\mu + \phi$, that would result from the neglected part of the sinusoidal function, is large and therefore makes only a slight contribution to the coupling process.

With the help of (30), we obtain from (29)

$$A_\mu(z) = \frac{a}{\sqrt{2}} K_{\mu s} A_s(0) \frac{e^{[i(\bar{\beta}_\mu - \beta_s - \phi) - (\alpha_s/2)]z} - 1}{i(\bar{\beta}_\mu - \beta_s - \phi) - \frac{\alpha_s}{2}}. \tag{31}$$

It can be shown that for lossless guides the relation $c_{\nu\mu} = -c^*_{\mu\nu}$ is required.[7] We use this relation in our present case since it must be approximately true even for lossy guides. We then obtain from (30)

$$c_{s\mu} = -\frac{a}{\sqrt{2}} K^*_{\mu s} e^{i\phi z}. \tag{32}$$

Substitution of (28), (31) and (32) into (27) yields

$$\alpha_s = \sum_{\substack{\mu=1 \\ \mu\neq s}}^{N} | K_{\mu s} |^2 \frac{1 - e^{[i(\beta_s - \bar{\beta}_\mu + \phi) + (\alpha_s/2)]z}}{i(\bar{\beta}_\mu - \beta_s - \phi) - \frac{\alpha_s}{2}} a^2. \tag{33}$$

In order to proceed further, we assume that mode s was inherently lossless prior to being coupled to the other modes. We also assume that the losses of the remaining modes are high. Since it appears reasonable to expect that α_s must be smaller than any of the loss coefficients of the other modes (these loss coefficients are the imaginary parts of $\bar{\beta}_\mu$) we can neglect the exponential term in (33) for large values of z so that we obtain

$$\alpha_s = \sum_{\substack{\mu=1 \\ \mu\neq s}}^{N} \frac{| K_{\mu s} |^2 a^2}{i(\bar{\beta}_\mu - \beta_s - \phi) - \frac{\alpha_s}{2}}. \tag{34}$$

The coefficient α_s is a complex quantity. Its imaginary part contributes only a slight change to the propagation constant of β_s. We are interested only in its real part. We write

$$\bar{\beta}_\mu = \beta_\mu - i\frac{\alpha_\mu}{2}. \tag{35}$$

In the denominator of (34), we neglect α_s compared to α_μ and obtain finally for the real part of α_s (which we write again α_s for simplicity)

$$\alpha_s = \frac{1}{2} \sum_{\substack{\mu=1 \\ \mu\neq s}}^{N} \frac{| K_{\mu s} |^2 a^2}{(\alpha_\mu/2)^2 + (\beta_\mu - \beta_s - \phi)^2} \alpha_\mu . \tag{36}$$

Equation (36) is the desired approximation. We assume that

$$|\beta_\mu - \beta_s - \phi| \ll \phi, \tag{37}$$

but require $\beta_\mu - \beta_s \neq \phi$ in the spirit of the nonresonant coupling assumption. Even for high loss modes we assume that the following relation applies

$$\alpha_\mu \ll \phi. \tag{38}$$

It is apparent that replacement of $\beta_\mu - \beta_s - \phi$ in the denominator of (36) with $\beta_\mu - \beta_s + \phi$ would lead to much smaller values of α_s. If we had used the sine or cosine function instead of the exponential function in (30), we would have obtained an additional term with $\beta_\mu - \beta_s + \phi$ (in the denominator) in (36). This additional term is much smaller than the leading term of α_s, so that our approximation (30) appears justified.

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